MATH 151 - FALL 2003

Answers to Sample Final Exam:

1. (a) \( \log R = a \log W + \log b \) where \( a = 1.8 \) and \( \log b = 34 \).
   (b) \( R = 34 \ W^{1.8} \)
   (c) \( 2^{1.8} = 3.5 \)

2. \( \lambda = 2 \) has eigenvector \( \begin{bmatrix} 1 \\ -1 \end{bmatrix} \) and \( \lambda = 4 \) has eigenvector \( \begin{bmatrix} 1 \\ 1 \end{bmatrix} \)

3. (a) \[
\begin{bmatrix}
8 & 16 & 25 \\
0 & 5 & 6 \\
2 & 9 & 7 \\
\end{bmatrix}
\]
   (b) \[
\begin{bmatrix}
9 \\
11 \\
\end{bmatrix}
\]
   (c) not defined

4. (a) \[
\begin{bmatrix}
J_{t+1} \\
A_{t+1} \\
\end{bmatrix} = \begin{bmatrix} 0 & 4 \\
.25 & 0 \end{bmatrix} \begin{bmatrix} J_t \\
A_t \end{bmatrix}
\]
   for the case in which you assume the sex ratio is 50:50 and females produce both male and female offspring. If you assume all 8 offspring are female, then the matrix becomes \[
\begin{bmatrix}
0 & 8 \\
.25 & 0 \\
\end{bmatrix}
\]
   (b) \[
\begin{bmatrix}
12 \\
1 \\
\end{bmatrix}
\]
   so it returns to the same in two time periods. If you use the second matrix above (with an 8 rather than a 4) you get \[
\begin{bmatrix}
24 \\
2 \\
\end{bmatrix}
\]
   (c) The eigenvalues are 1 and -1 for the first case, so long term growth rate is 1, but this means the population actually oscillates every two time periods returning to the same structure, with a ratio of 12:1 juveniles to adults as at the start. For the second matrix, the eigenvalues are \( \sqrt{2} \) and \(-\sqrt{2}\) and the long term ratio of juveniles to adults is \( 4 \sqrt{2} : 1 \).

5. (a) .9 (b) .6 (c) .1

6. (a) 1/16 (b) 1/28

7. (a) .52 (b) 1/13 (c) 1/3

8. (a) \( x_n = (3.5) \ 3^n + .5 \)
   (b) \( x_n = 5n + 2 \)
   (c) \( x_n = (2) \ 3^n - 4 (-1)^n + 4 \)
   (d) \( x_n = (2) \ 3^n - 2^n + 4 \)
9. \(1 - P[\text{No 5's at all}] = 1 - \frac{125}{216} = .422\)

10. (a) \(x_n = x_0 (1.3)^n\)
    (b) sometime during the 4th day - solving gives \(n = 4.19\), so if you check only daily, the tripling will be observed on day 5.

11. (a) \(\frac{1}{3}\)  (b) \(\frac{1}{16}\)  (c) not defined  (d) \(\sqrt[3]{3}\)