EEB 610: Homework 2

Due Thursday Oct. 18th

Name: _____

- Please show your work when answering a question.
- Don't waste your time typing your answers, legible handwritten responses are fine.
- Our expectation is that this homework will take six to eight hours to complete.
- You are encouraged to help each other and discuss the problems in the homework, however, what you turn should be your own work.
- If you are having problems, getting stuck on a particular point, or don't understand a question, please contact either instructor.

Logistic equation

1. On the same axes, plot the population of the US by the the exponential growth, and the continuous logistic model since 1850 to date. Provide the graph.

   (a) Fit both models to the data using a least squares approach (i.e. find the parameter values, K and/or r, that minimize $SS = \sum(N(t_i) - x_{t_i})^2$ where x_t is the data at time t do this using the built in 'Solver' function in Excel. See References below for more instruction). The model with the lower SS value is a 'better' fit.

      i. Which model do you think fits the data the best? Why?

      ii. If $SS_{\text{expo}} - SS_{\text{logistic}} > 1.92$ then, statistically speaking, the data supports the idea of a carrying capacity at the $p < 0.05$ level. Is this condition met?

   (b) A part of this exercise is to find the population of the US from 1850 to date. In the data, there are a few anomalies. With your knowledge of US and world history, rationalize the stronger changes in the population dynamics. Hint for part of the data:
   `http://www.census.gov/popest/archives/1990s/popclockest.txt`

2. For the discrete logistic equation, we did not get to finish the analysis of the 2-periodic solution that occurs when $\alpha > 3$. In this exercise, you are asked to show

analytically that there is a 2-periodic solution for the equation $z_{t+1} = \alpha z_t[1 - z_t]$. Some suggestions about the route to the answer follow:

(a) Find z_{t+2} in terms of z_t.

(b) Now find the equation that the equilibria satisfy. This is a cubic equation.

(c) Observe one equilibrium is $z^* = 1 - \alpha^{-1}$ (why should this be the case?) so the other two satisfy a quadratic equation that when $\alpha > 3$ has both roots real. What are these two equilibria?

(d) Which equilibria form the 2-periodic solution?

(e) Pick an α in $(3, 1 + \sqrt{6})$ and show that your two equilibrium solution is "stable" for at least two initial values "close" to the 2-periodic solution. Do this in Excel and provide the graphs.

(f) For α as above, discuss the stability of the equilibrium in (c) above.

i. Please argue this from our previous discussion about the stability of this solution.

ii. Using Excel provide a graph and convince yourself (and us) that your conclusion in (f)i. is correct.

3. The theta logistic model is a generalization of the standard logistic equation. The theta logistic is defined as

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)^{\theta}.$$

Below you will examine the behavior of the model under the parameter values of theta $r = 0.25, K = 1$ and $\theta = 1/2, 1$ and 2.

(a) Plot $\frac{dN}{dt}$ vs N and describe how changing θ affects how the growth rate of the population changes with N.

(b) Analytic solutions for this model exist only in restricted cases of θ and are quite complex. So instead of finding the explicit solutions for $N(t)$, you are encouraged to solve the model numerically by updating the equation using very small, but discrete time steps.

Plot $N(t)$ vs t for the different parameter values. Describe the temporal dynamics of the system and explain how this behavior can explained using the previous plot of $\frac{dN}{dt}$ vs N.

References

Links for using Solver function in Excel to minimize the Sums of Squares:

- http://www.chem.mtu.edu/~fmorriso/cm4650/Using_Solver_in_Excel.doc
- http://web.mit.edu/5.310/www/Kinetics_F05_Appendix_3.pdf.