**Drug testing Example for Conditional Probability and Bayes Theorem**

Suppose that a drug test for an illegal drug is such that it is 98% accurate in the case of a user of that drug (e.g. it produces a positive result with probability .98 in the case that the tested individual uses the drug) and 90% accurate in the case of a non-user of the drug (e.g. it is negative with probability .9 in the case the person does not use the drug). Suppose it is known that 10% of the entire population uses this drug.

You test someone and the test is positive. What is the probability that the tested individual uses this illegal drug?

What is the probability of a false positive with this test (e.g. the probability of obtaining a positive drug test given that the person tested is a non-user)?

What is the probability of obtaining a false negative for this test (e.g. the probability that the test is negative, but the individual tested is a user)?

Let:
+ = the event that the drug test is positive for an individual
- = the event that the drug test is negative for an individual
A = the event that the person tested does use the drug that is being tested for

We want to find: $P[A\,|\,+]$, $P[+\,|\,\bar{A}]$ (false positive), and $P[-\,|\,A]$ (false negative)

We know that $P[A] = .1$, $P[+\,|\,A] = .98$, and $P[-\,|\,\bar{A}] = .9$ and from this we know that $P[+\,|\,\bar{A}] = .9$ which allows us to find using Bayes Theorem:

$$P[A\,|\,+] = \frac{P[+\,|\,A]\;P[A]}{P[+]}$$

so

$$P[A\,|\,+] = \frac{P[+\,|\,A]\;P[A]}{P[+\,|\,A]\;P[A] + P[+\,|\,\bar{A}]\;P[\bar{A}]}$$

or

$$P[A\,|\,+] = .98\;\frac{(.1)}{.98\,(.1) + .1(.9)} = .52$$

Also, the probability of a false positive is

$$P[+\,|\,\bar{A}] = 1 - P[-\,|\,\bar{A}] = .1$$

and the probability of a false negative is

$$P[-\,|\,A] = 1 - P[+\,|\,A] = .02$$