Title: Cycling in the Complexity of Early Societies

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Abstract:

Warfare is commonly viewed as a driving force of the process of aggregation of initially independent villages into larger and more complex political units that started several thousand years ago and quickly lead to the appearance of chiefdoms, states, and empires. Here, we build on extensions and generalizations of Carneiro’s (1970) argument to develop a spatially explicit agent-based model of the emergence of early complex societies via warfare. In our model polities are represented as hierarchically structured networks of villages whose size, power, and complexity change as a result of conquest, secession, internal reorganization (via promotion and linearization), and resource dynamics. A general prediction of our model is continuous stochastic cycling in which the growth of individual polities in size, wealth, and complexity is interrupted by their quick collapse. The model dynamics are mostly controlled by two parameters, one of which scales the relative advantage of wealthier polities in between and within-polity conflicts, and the other is the chief’s expected time in power. Our results demonstrate that the stability of large and complex polities is strongly promoted if the outcomes of the conflicts are mostly determined by the polities’ wealth, if there exist well-defined and accepted means of succession, and if control mechanisms are internally specialized.

Introduction

For most of humanity’s existence, people lived in small egalitarian bands or villages which were politically autonomous. However, several thousand years ago a qualitative change happened with villages starting to aggregate into larger and more complex polities (a general term that includes not only states or empires but also smaller-scale independent political units, such as chiefdoms, acephalous tribes, and autonomous villages, e.g. Feguson and Mansbash [Type text]
structured hierarchically. The process of aggregation lead, over time, to the emergence of chiefdoms, states, and empires. Once established, these complex societies rose and fell over time, with centers of power and authority shifting from one location to another over the landscape, a process that has been described as cycling (Wright, 1977, 1984; Cordy 1981; Kirch 1984; Marcus, 1992, 1998; Anderson, 1994, 1996; Earle 1997; Cioffi-Revilla and Landman 1999, Junker, 1999; Hall, 2001, and whose causes have fascinated scholars and been the subject of speculation for centuries (Engels, 1884; Childe, 1950; Wittfogel, 1957; Adams, 1966; Fried, 1967; Flannery, 1972; Webster, 1975; Wright, 1977, 1984, 1986; Service, 1978; Ferguson and Mansbach, 1996; Earle, 1997; Trigger, 2003; Drennan and Peterson 2006; Turchin and Gavrilets, 2009, Spencer 2010).

Here we are concerned with a set of influential theories that put special emphasis on warfare between different polities (starting with villages). When warfare first occurred in human (pre)history is controversial, although it is assumed to have been relatively small in scale and consequence until complex and presumably multicommunity societies emerged (Ferguson, 1984; Haas, 2004; Trigger, 2003; but see Keeley, 1997). Besides warfare, there are of course a number of additional prerequisites for the evolution of social complexity. One, emphasized by Carneiro is circumscription (environmental, due to the resource concentration, and social, due to the presence of other human groups nearby, Carneiro 1970, 1981). Circumscription was the factor that precluded losing communities from moving away, and thus separating themselves spatially and politically from the victors. Other prerequisites include the existence of agricultural potential capable of generating surpluses and a significant variation in productive and/or demographic potential between local communities (Webster, 1975). Equally important was the ability to delegate power and the invention of hierarchically structured control mechanisms in which each superior directly controlled only a limited number of subordinates (Flannery, 1972; Wright, 1977, 1984; Turchin and Gavrilets, 2009). The latter was also important for the subsequent growth of polities given what has been called “scalar stress”, a decrease in the ability of leaders to process information and maintain efficient control over subordinates as their number (herein, the number of subordinate villages) increased (Johnson, 1982). The outcome of these processes and factors was the emergence of simple chiefdoms (Steponaitis, 1978, 1981; Wright, 1984) in which one village controlled (and received tribute from) several subordinate villages it had previously defeated. More complex polities were characterized by greater numbers of subordinate levels, with complex chiefdoms, paramount chiefdoms, and state societies typically defined as those polities with two, three, and four or more administrative levels above the local or primary community, respectively (Flannery, 1972; Wright and Johnson, 1975; Steponaitis, 1978; Wright, 1984; Anderson, 1994).

The paramount chief delegated the power over a subset of his villages to somebody else (a subchief), often a relative (e.g. Cordy 1981). Sometimes the chiefs of vanquished groups were permitted to stay in power but had to pay tribute (e.g. Kurella 1998). The hierarchical nature of this organizing principle allows, in theory, for unlimited growth in the size and complexity of chiefdoms. However, in early chiefdoms, the constituent communities can exist autonomously. Moreover, in these societies control was vested in one or a few individuals, and such absence of internal specialization meant that subchiefs had almost total control over their subordinate villages (Wright, 1977, 1984; Earle, 1987). Therefore rebellion and secession by subchiefs had a relatively low cost and was relatively easy to organize, if not always successfully accomplished. As a result, the growth in the size and complexity of chiefdoms was counterbalanced by a tendency to fragment through rebellion.
Although the argument just given is well accepted by anthropologists and historians, many questions remain. These concern the levels of complexity that can be achieved, its dynamic patterns and timescales, and the qualitative and quantitative effects of various parameters and factors. Here we use a stochastic spatially-explicit agent-based mathematical model to shed light on these questions. The analyses that follow encompass developments over large geographic and extended temporal scales, the processes that cause chiefdoms, states, and empires to emerge, persist, and collapse at the scale of decades to centuries, the longue durée of human history. Our approach is a generalizing one, sacrificing specific detail for a glimpse of the reasons behind the broad patterns recorded by archaeology and history. At the same time, however, our modeling approach aims to connect these broad processes to the finer scale historical events generating the patterns under examination.

Until recently there has been only a limited amount of modeling work directly addressing the evolution of large-scale polities (Dacey 1969, 1974; Bremer and Mihalka 1977; Cusack and Stoll 1990; Cederman 1997, Spencer 1998; Cioffi-Revilla 2005; Cederman and Girardin 2010). Most of this work has focused exclusively on polity size, was limited to a small number of simulation runs, and was primarily motivated by questions of interest to political scientists. In contrast, we present a dynamic quantitative model exploring the origin and operation of early human complex society, that focuses on both the size and complexity of emerging polities as well as their longevity and settlement patterns. We systematically examine the effect of parameters such as system size, the scaling of polity power to the probability of winning a conflict, tribute level, variation in productivity between individual villages, span of control, and chief’s average time in power. The polities in our model exhibit a strikingly fluid nature resembling so-called “chiefly cycles”. Unexpectedly, the largest effect on results is due to just two parameters: the scaling of the polity power to the probability of winning a conflict, and the chief’s average time in power. At the end of the paper we discuss the implications of our results and some relevant empirical evidence. Some preliminary results of our model were presented in Turchin and Gavrilets (2009).

Model

Here we describe the model informally; readers interested in the mathematical details will find them in the Mathematical Appendix. We consider a hexagonal array of initially autonomous local communities (villages). Each community is represented by a hexagon and has up to six neighbors (Haggett, 1965). Time is discrete and the unit of time (“year”) is the expected interval between two consecutive “decisions” made by a community (explained below). Each community $i$ is characterized by a constant base-line resource level $f_{0i}$ which can be interpreted as a measure of the settlement’s catchment size (Steponaitis, 1981). The values are chosen randomly and independently from a (truncated) normal distribution with mean 1 and constant standard deviation $\sigma$. Parameter $\sigma$ measures the variation in productive/demographic potential between local communities due to the heterogeneity of the environment. Each community is also characterized by the actual resource level. Initially, for each community the actual resource level is set at the base-line level (i.e., $f_{0i}$), but different actions in which the community takes part change its value (explained below).

Each community is a part of a polity (which can consist of a single community). The polities have a hierarchical structure. Each community in a polity except for the one at the top of the hierarchy (the “chief community”) has one superior community and may have up to $L$ subordinate communities, where $L$ is a constant parameter measuring the maximum span of control (i.e. the maximum number of subordinates; Johnson, 1982). The polities are identified [Type text]
by their chief communities (see Fig. 1). Each subordinate community pays a tribute by transferring a fixed proportion $\theta$ of its total resources to its superior. The total resources of a community are the sum of the resources it produces and the tribute received from subordinates (Steponaitis, 1981). The power (wealth) of the polity is the total resources controlled by its chief community. The complexity of the polity is the number of levels of control above the level of individual villages.

The polities are engaged in warfare. The polities grow, decrease in size, or disappear as a result of conquest, with the polity-winner absorbing (all or a part of) the polity-loser. New polities also appear, and old polities decrease in size, when a subordinate community secedes from its polity with all of its subordinates.

Each chief community and each of their direct subordinates make exactly one decision every year. For the chief community, the decision is whether or not to attack a neighboring polity. For a direct subordinate of a chief community, the decision is whether or not to attempt to secede.

Warfare is modeled as follows. A polity selects its weakest neighbor and calculates the chance of success of an attack upon it (which increase the probability of attack), as well as the attack costs (which decrease the probability). The willingness to attack also decreases as the amount of resources available decreases. An attack of polity $i$ on polity $j$ succeeds or not with probabilities proportional to $\theta_i$ and $\theta_j$. Parameter $\alpha$ characterizes the importance of other factors (“noise”) besides the polities’ power in controlling the outcome of a conflict, with larger $\alpha$ implying less noise and more determinism. For example, let polity $i$ be twice as strong as polity $j$. Then with linear scaling (i.e., with $\alpha=1$), the probabilities of polities $i$ and $j$ winning the conflict between them are in the ratio 2:1. However with quadratic scaling (i.e., with $\alpha=2$) this ratio becomes 4:1. That is, as $\alpha$ increases, polity strength becomes a better predictor of the outcome of conflict.

The aggressor attempts to conquer communities of the victim, starting with border ones, and proceeding in a series of “battles” until either it suffers a defeat, or until the chief community of the victim polity is conquered. Thus, the aggressor either fails completely, seizes a part of the victim polity, or the whole victim polity is annexed.

Annexing communities may require reorganization of the successful aggressor polity (via linearization and promotion, Flannery 1972), because of the limit $L$ on the number of subordinates of any community. Thus, if one community is to become a subordinate of another, the latter must have at least one open control slot. When all open slots are exhausted, new ones are created by demoting some communities, i.e., moving them to a lower level in the hierarchy (Flannery, 1972). The winning polity attempts to maximize the flow of tribute to the top, and therefore demotes poorer/smaller communities while keeping wealthier/larger ones at higher levels of the hierarchy.

A community subordinate to the chief polity will secede if it estimates that the attack of its old master will be successfully repelled and is willing to pay the price of rebellion. The chief polity attempts to suppress the rebellion immediately. If a successful rebellion results in spatial separation between different parts of the master state, all communities that become disjointed from their superiors secede as well. To account for a possibility of secession upon the death of the paramount chief as a result of a struggle among subchiefs (which is a major source of instability in chiefdoms, see Anderson, 1994; Wright, 1984; Cordy 1981; Kirch 1984), we introduce an additional parameter $\tau$, the average time in power of the paramount chief. Upon the death of the paramount chief, a random number of subordinate communities become independent without war.
The cost of warfare is a reduction in the amount of actual resources available to participants, with less likely outcomes being costlier for all participants. Following conflict resolution resources are renewed at a fixed low rate.

**Analysis**

To get intuition about the model’s behavior, we ran numerical simulations with all possible permutations of the following six parameters: system edge size $S=4$, 5 and 6 villages (so that the total number of villages is 37, 61, and 91, respectively); $\alpha=1$ and 2 (i.e., linear and quadratic scaling of the polity power to the probability of a win); variation in productivity $\sigma=0.3, 0.4$ and 0.5 (using data in Steponaitis, 1981, $\sigma$ can be estimated to be between 0.34 and 0.48), tribute $\theta=0.1, 0.2$ and 0.3 (in Steponaitis, 1981, tribute level was estimated to be 0.16-0.22), span of control $L=5, 6$, and 7 (Johnson, 1982, argued that the most common value of the span of control is 6), and the chief’s average time in power $\tau=5, 10$ and 20 years. Numerous sources, from Polynesian chiefly genealogies to the so-called “king lists” of many early states indicate these are viable parameters. Very few leaders in chiefly or even state level societies lasted longer than 20-30 years, with most reigns appreciably shorter; rulers who held power for exceptionally long times are just that, unusual exceptions rather than the rule (Kamakau, 1872; Beckwith, 1977; Dodson and Hilto, 2004).

Each simulation ran for 1000 years, and the statistics were evaluated using the data from the last 800 years. Our focus was on the dynamics of the relative size of the largest polity $s_{\text{max}}$ (Fig. 2a), the mean $\bar{c}$ (Fig. 2b) and maximum $c_{\text{max}}$ complexity, and the average “centrality” $P$ (i.e., the ratio of the power of the chief village and the one immediately below, Steponaitis, 1981) (Fig. 2c). We also looked (see Supporting Information) at the relationships between a polity’s base-line productivity and actual power (Steponaitis, 1981) and between settlement power and rank on the log-log scale (Johnson, 1980; Wright, 1984), and at the distributions of village power (Wright, 1984).

Starting with a system of independent villages, we observe the rapid formation of polities of various size and complexity as a result of warfare. The system quickly (within 50-100 years) reaches a kind of equilibrium in which our focal characteristics $s_{\text{max}}, \bar{c}, c_{\text{max}}$, and $P$ are maintained at approximately constant values (e.g. Fig. 2). However this equilibrium is stochastic and is characterized by the dynamic instability of the individual polities, with quick collapse characterizing chiefdoms reaching relatively large size and complexity. To quantify this process, we identified all “significant complex chiefdoms”, i.e. the polities with complexity $c \geq 2$ and size $s \geq 10$ villages. Note that only a small proportion of polities reach this status. Figure 3, illustrating the dynamics of such polities, shows their quick growth and collapse.

We studied the effects of parameters on the system properties (see Table 1 and SI). The relative size of the largest polity $s_{\text{max}}$ increases most significantly with the success probability exponent $\alpha$ and with the chief’s average time in power $\tau$ but decreases with the system size $S$ (see Fig.4 and SI). With $\alpha=2$ (i.e., with quadratic scaling of the polity power to the probability of a success) we occasionally observe cases when all villages are incorporated in a single polity. Such a state can last for up to 35% of the run time and is most likely with the maximum values of both $\tau$ and $\theta$.

The average complexity $\bar{c}$ increases most significantly with $\alpha$ and $\tau$. It also increases with system size $S$, but decreases with increasing span of control $L$. Overall, $\bar{c}$ stays below ca. 2 and 3.3 for $\alpha=1$ and 2, respectively.
The average centrality $\mathcal{P}$ increases most significantly with variation in productivity $\sigma$ and with tribute $\theta$; it also increases with $\alpha$ but decreases with $\tau$.

The average lifetime of ‘significant complex chiefdoms’, $T$, increases with $\alpha$ and $\tau$ (most dramatically). It also grows with tribute $\theta$ but decreases with system size $S$. Overall, the average lifetime of the 10 most significant complex chiefdoms stays below 55 and 68 years for $\alpha=1$ and 2, respectively.

The rank-size curves describing the distribution of polity sizes (Haggett, 1965; Johnson, 1980; Wright, 1986; Peterson, and Drennan 2005; Drennan and Peterson 2006) are always convex (see Fig.5); polity power declines approximately linearly with the logarithm of its rank indicating the presence of poorly integrated competing centers. The scatter plots for the relationships between the actual and base-line power of polities (Steponaitis, 1981) do not show much clustering, suggesting that they are a poor indicator of the degree of complexity in the system (see SI).

**Discussion**

Our model provides theoretical support for a view that the formation of complex polities is “a predictable response to certain specific cultural, demographic and ecological conditions” (Carneiro 1970). The conditions explicitly accounted by our model include warfare, circumscription, variation in productivity between different local communities, the ability to generate surpluses, the ability to delegate power, and restrictions on the growth of polities due to scalar stress. Once these conditions are present within a particular geographic area, the model predicts rapid formation of hierarchically organized competing polities partitioning available space.

A striking feature of the model output is the fluid nature of ‘significant’ polities, which continuously go through stochastic cycles of growth (both in size and complexity) and collapse. Growth is driven by successful warfare whereas collapse results from defeat in warfare, the rebellion of subchiefs, and fragmentation following the death of the paramount chief. The lifetime of chiefdoms observed in our simulation - a few of generations - is comparable to those identified by archaeological studies (e.g., Anderson 1994; Wright 1984; Earle 1991; Hally 1996; Junker 1999; Blitz and Livingood 2004). The model suggests that the rapid collapse of chiefdoms can occur even without environmental perturbations (e.g. drought) or overpopulation.

While the characteristics of individual polities (such as size, complexity, power, and centrality) undergo continuous change, the average values of these characteristics across the whole system remain relatively stable. We have systematically studied how these characteristics are affected by the following six parameters: variation in productivity between local communities $\sigma$, probability of success in war exponent $\alpha$, span of control $L$, tribute $\theta$, system size $S$, and the average chief’s time in power $\tau$. Our results show that most variation in system behavior can be explained just by two parameters: $\alpha$ and $\tau$, with higher values strongly promoting the existence of larger, more complex, and more stable polities. Only in the case of centrality were the effects of $\alpha$ and $\tau$ small, with most variation being explained by $\sigma$ and $\theta$.

The chief’s expected time in power $\tau$ is one of two most important parameters. This finding strongly supports arguments on the crucial importance of having well-defined and accepted mechanisms of succession for the stability of polities (Anderson, 1994; Wright, 1984). Creating and maintaining complex polities thus requires having effective mechanisms in place to deal with both internal and external threats; the former were most notably associated with factional competition and the succession/replacement of leaders, the latter with the agendas
and actions of leaders in other polities. Even a most abbreviated reading of human history shows how difficult this task has been.

The other critical parameter of the model is the probability of success in war exponent \( \alpha \), which controls the relative effectiveness of stronger (wealthier) polities in within- and between-polity conflicts. In our model, the stronger of the two polities does not necessarily win a conflict between them. This is reasonable as there are many other factors besides the wealth that can affect the outcome of conflict. However increasing \( \alpha \) implies a stronger dependence of the outcome of the conflict on the polities’ power (wealth). The degree of determinism in the conflict resolution (and thus, parameter \( \alpha \)) is expected to increase with the economic and political development of polities (Carneiro, 1970, 1981; Collins, 1986). Note that in our simulations, the polities conquering the whole simulation domain are observed only with \( \alpha=2 \).

Our model shows no qualitative differences between polities with a single level of control above the level of individual villages ("simple chiefdoms") and polities with two or more levels of control ("complex chiefdoms" or "states"). During each individual run, the number of control levels is not stable but changes dynamically and therefore cannot by itself serve as an indicator of the presence of “true” states. Our results support Carneiro’s (1981) insight that “the transcending of local sovereignty and the aggregation of previously autonomous villages into chiefdoms was a critical step in political development - probably the most important one ever taken. It crossed a threshold, and once crossed, unlimited further advance in the same direction became possible. The emergence of chiefdoms was a qualitative step. Everything that followed, including the rise of states and empires, was, in a sense merely quantitative” (p.38).

In our simulations, it is possible for polities to conquer the whole simulation domain, or a significant part of it, but our analyses also show that such polities are relatively short-lived. A major reason for this is the relative ease of rebellion. Additionally, our model explicitly assumes that any “internal specialization” is absent and that all mechanisms for the autonomous existence of a rebellious province are already in place. This model behavior thus further emphasizes, through the effect of its absence, the importance of “internal specialization” for the emergence of large and stable polities (Flannery, 1972; Wright, 1977, 1984).

Archaeological analyses documenting settlement hierarchies by nature of the coarse grained temporal resolution available typically combine sites occupied over intervals of a half century or more to hundreds of years. The hierarchies recognized by archaeologists are commonly displayed as a series of maps showing site sizes during different periods, often separated by a century or more, or else bar graphs or rank size curves (Wright and Johnsonn 1975; Wright, 1977, 1984, 1986; Johnson, 1980; Hally 1996; McAndrews, et al., 1997; Spencer and Redmond, 2001; Liu and Chen, 2003; Peterson, and Drennan 2005; Drennan and Peterson 2006). All of these reconstructions suggest rigid formal hierarchies and a static if not fairly stable political landscape. Our analyses, in contrast, indicate that at a finer temporal scale the various factors that produce these archaeological signatures are far more dynamic. This result is in agreement with written records of historical events.

Our estimates of chiefdom duration are comparable with those based on archaeological evidence. In studying Southeastern Mississippian chiefdoms Hally (1993) examined the time periods when occupation and mound construction occurred at 47 mound centers in central and northern Georgia. He concluded that “paramount chiefdoms must have been unstable and short lived” while “simple and complex chiefdoms endured for as much as a century or more” (Hally 1993). The actual duration of phases, or periods of occupation and construction in his analysis (p. 145), however, could not be resolved much below 75-100 years. Hally followed
this analysis up in a second paper (Hally 1996) examining 45 mound centers, and including
episodes of mound stage construction. Where evidence for numbers of internal mound
construction stages was available, duration of occupation was estimated to be between 75-100
years, with the average number of years per stage ranging from 12 to 25 years at the best
understood sites (Hally 1996). This span may represent the duration of a chiefly leader, or
generation. At 29 of the 45 mound centers, only a single period of use is currently known,
indicating most “chiefdoms” locally lasted no more than 75-100 years, and perhaps
appreciably less, given that the number of construction episodes within many mounds was
only rarely more than 1 or 2 which, based on the better documented centers, likely went up in
10-20 years or less (Hally 1996).

In a follow up, Hally  (2006:27) argued that “as many as 47 distinct chiefdoms rose and
fell” in 27 locations during the Mississippian period in northern Georgia (some locations were
occupied repeatedly, often with gaps in occupation of a century or more). The numbers of
chiefdoms in his sample changed century by century from AD 1000 to AD 1500, from 10 to
13 to 8 to 11 and 17 (Hally 2006). Many chiefdoms in the sample were single mound, or
simple forms (Hally 2006).

Blitz and Livingood (2004) used mound volume as an alternate means of measuring
regional settlement hierarchies, using a sample of sites from across the southeast. Looking at
35 mounds, they recorded a mound volume index (basal length x basal width x height/1000),
the number of major mound-construction stages, the duration of mound use in years, and the
number of mounds at the site where the sample mound was found (p.293). Their analysis,
while geographically broader than Hally’s, yielded generally similar results, noting average
mound center “duration of use range is 100-450 years, with a mean of 183 years and a median
of 150 years. Also, there appears to be a rough periodicity in mound construction: the average
occupation span per construction layer is 25-50 years.” (p. 296). They were able to
demonstrate that mound stage construction might fall into two cycles, one of ca. 12-18 years,
and another of ca. 25-50 year spacing (p.297). They also found that mound construction at
major centers was appreciably different in scale and volume than at minor centers (p. 298).
That is, they argued that care must be taken in directly equating mound stages with specific
types of behavior (i.e., chiefly succession), since construction differed between large and
small centers.

Similar results suggesting reasons for the duration and stability of polities have been
obtained in other parts of the world where the dynamics of chiefdoms and states have been
examined. Cioffi-Revilla and Landman (1999), for example, analyzed data on 72 Maya
primarily state-level polities existing over a time span of 2500 years with respect to variables
characterizing their duration and stability; their analysis, it should be noted, focuses on the
total duration of these polities, and not on fluctuations in occupation at specific centers that
characterize the southeastern examples noted previously. Nonetheless, even though the scales
are different, the results are very instructive. Their data show two distinct cycles of political
development, a "major cycle" or steady increase in the number of polities together with sharp
reversals from ca. 900 B.C. to A.D. 1200, and a "minor cycle" characterized by the formation
of a much smaller number of larger subregional states from ca. A.D. 1200 to 1700. The
duration of polities varied significantly between those polities founded during the relatively
calm Preclassic period from 900 B.C. to A.D 250 (on average 1,154 +/- 96 years); those
founded during the more interactive and competitive Classic period from A.D. 250 to 900 (an
average 348 +/-24 years; and those during the Post-Classic from ca. A.D. 900 to 1530, which
were of even shorter duration (218 +/- 42 years). Cioffi-Revilla and Landman (1999) analysis
supports a model of Maya political dynamics based on Preclassic origins, punctuated phases
of development, multiple cycles of system expansion and collapse, and weaker political
[Type text]
stability for increasingly complex polities. According to Cioffi-Revilla and Landman (1999) the major reason for the Maya political collapse at the end of the Classic period and after was that a pan-Maya level of political integration or unification failed to develop; such a framework, they argue would have been necessary to sustain the growing number of polities already containing millions of inhabitants.

Our finding that the duration of a chief’s reign is a significant parameter parallels that in the literature on state fiscal organization. In this literature, the discount rate of rulers (that is, their expected time in office) is shown to be a major determinant of the kind of taxation system employed, which in turn has various implications for society, e.g. for political stability of the Roman state and Ptolemaic Egypt (Kiser, and Kane 2007; Levi 1988; Monson 2007).

In our model, which we believe closely mirrors events that occurred in many parts of the world, individual villages differ only in their base-line productivity and geographic location but otherwise have equal ability to form complex societies. In human history some polities had a headstart, allowing them to achieve large size initially; but the strategies for complex polities buildup and maintenance would have spread quickly in a Darwinian fashion as a result of conquest and imitation. Therefore once chiefdoms appeared, their organizational form would itself have tended to spread, as neighboring societies adopted it for reasons ranging from emulation to self defense (Carneiro, 1981; Anderson, 1994).

The dynamics generated by our model, in which hierarchical societies tend to achieve at most medium levels of complexity, and only for relatively short periods of time, resembles the chiefly cycles observed prior to sustained Western contact in eastern North America, southern Central America and northern South America, Oceania, southeast Asia and the Philippines, and across large parts of sub-Saharan Africa (e.g., Wright, 1984; Marcus, 1992; Earle, 1991; Anderson, 1994, Cordy 1981; Junker 1999; Drennan and Peterson 2006).

The model developed here can be extended in a number of ways. For example, instead of a simple conquest mechanism, one can consider a more nuanced dynamic in which external threat of conquest (or raiding) induces a greater degree of cooperation between lower-level groups, which results in a more stable higher-level polity. One possible direction is to generalize the model to allow for the formation of coalitions between different polities (Carneiro 1998; see Gavrilets at al. 2008 for a possible dynamic approach). Also, to adapt the model for describing larger spatial scales (e.g., as necessary for modeling the origin of states and empires), changes in population densities need to be considered, as well as the propensity for cooperation (and, conversely, conflict) should be allowed to depend on cultural similarity/dissimilarity between the agents.

Over the past several decades mathematical methods and techniques have become very important in life sciences and social sciences (Spencer, 1998; Cioffi-Revilla 2002; Bentley and Maschner, 2008; Costopoulos, 2008; Kohler, et al., 2005). In particular, mathematical modeling is a powerful tool for better understanding of the origins of new species (Gavrilets, 2004) and of general rules of biological diversification (Gavrilets and Losos, 2009). Modeling efforts like those advanced here offer fruitful avenues for future research on general patterns in historical dynamics and on the emergence and diversification of human societies (Turchin, 2003, 2006).

**Mathematical Appendix**

Here we provide some additional details on the model and simulations.
**Attacks.** A polity may attack only its weakest neighbor. The attack of polity $i$ on polity $j$ is successful with probability

$$P_{ij} = \frac{F_i^\alpha}{F_i^\alpha + F_j^\alpha}$$

where $F_i$ is the power of polity $i$, and $\alpha$ is the success probability exponent. Polity $i$ will attack polity $j$ only if it estimates that the attack will be successful, is willing to pay the cost of warfare, and is not too devastated by previous warfare. Specifically, the probability of attack is set to

$$A_{ij} = P_{ij} \exp(-\beta c_{ij}) \frac{F_i}{F_{i,0}}$$

Here, the first term is the probability of winning (estimated by the potential aggressor via “scenario building”). The second terms accounts for a negative effect of costs of warfare, (defined below), on the willingness to attack; $\beta>0$ is a parameter. The third term accounts for a reduction in the willingness to attack caused by recent conflicts (and ensuing drop in available resources); $F_{i,0}$ is the maximum possible power of the $i$-th polity (observed at maximum possible level of resources, i.e. if all). If attack is not successful, a war ends in a draw.

We assume that attacks proceed through one or more stages. At the first stage, the target is the wealthiest border community of the weakest neighbor. The victim repels the attack successfully with probability $\gamma$. If the attack was successful, the aggressor proceeds to attack the superior of the target. Now the probability that the victim repels the attack successfully is reduced to $\gamma$, where $0<\gamma<1$ is a parameter characterizing the “loser effect” (e.g., due to demoralization). If the second attack is successful, the aggressor proceeds to attack the superior of the superior of the target and so on. The process stops when an attack is repelled or when the chief community of the victim polity is conquered. In the former case, the aggressor seizes a part of the victim polity that was subordinate to the community attacked at the last successful attack. In the latter case, the whole victim’s polity is seized.

**Linearization and promotion.** Polity $i$ attempts to maximize the flow of tribute by the processes of linearization and promotion, after Flannery (1972), subject to geographic restrictions and restrictions on the number of subordinates. If the chief community $i$ has an open control slot, it will control polity $j$ directly. If there are no open control slots, then the chief community will control directly $L$ wealthiest communities chosen (i.e., promoted) from the set of its $L$ subordinates and the newly conquered polity $j$. The remaining (i.e., the poorest) community will be demoted and reattached to its geographically closest neighbor of the higher rank (i.e., by-passed in a process akin to linearization). If this neighbor has already filled all its control slots, further rearrangements will follow according to the same strategy.

**Costs.** Different actions (i.e., attack, defense, rebellion, or suppression of rebellion) reduce the actual resource level for all participants by a factor $(1-c)$ where the cost $c$ of an action is equal to a constant $\delta$ times the probability of loss for the winner $(0<\delta\leq 1)$. That is, if is the probability that an attack of polity $i$ on polity $j$ is successful, then the cost of a successful attack is

$$c = \delta(1-P_{ij})$$

(3a)

whereas the cost of an unsuccessful attack is

$$c = \delta P_{ij}.$$

(3b)
This simple model captures the idea that more likely outcomes are less costly to all participants. For attacks involving several stages, costs are combined multiplicatively.

**Resource dynamics.** Each year the actual resource level grows towards its baseline level at an exponential rate. Specifically, we define the half-life of resource recovery $r$ measured in years so that it takes $r$ peaceful years for the resource to grow from $1−\delta$ to $1−\delta/2$.

**Implementation rules.** We use a “parallel” implementation of the model in which different actions happen simultaneously rather than sequentially. To handle multiple events potentially involving the same polity we use the following rules: 1) A polity that is subject to a rebellion does not attack other polities. 2) A polity that is subject to a rebellion is not attacked by other polities. [The justification: since dealing with the rebellion will make the polity weaker, potential attackers would prefer to wait and attack later.] 3) If there are multiple rebellions within a polity, the polity’s power is divided proportionally and multiple suppression attempts occur simultaneously.

**Acknowledgements.**
We thank the reviewers for valuable comments and suggestions. SG was supported by a Guggenheim Fellowship.

**References**


**Figure Legends**

Fig.1. An example of a state of a system with 37 villages. There are four polities. (a) Spatial view. The arrows point the direction of the tribute flow. The circles are proportional to the polity power. The numbers are labels identifying the chief communities. (b) A hierarchical representation of the polities. The complexity of polities 3, 26 and 30 is 2 while that of polity 17 is 1.

Fig.2. Examples of the temporal dynamics of the relative size of the largest polity, the mean complexity, and the mean centrality. Blue lines: $S=5, \alpha=1, \sigma=0.4, \theta=0.2, L=6, \tau=10$; green lines: same but with $\alpha=2$.

Fig.3. The dynamics of the sizes of all polities that have achieved a size of at least $s=10$ and a complexity of at least $c=2$. Each color is unique for a particular chief village. (a) $S=5, \alpha=1, \sigma=0.4, \theta=0.2, L=6, \tau=10$; (b) same but with $\alpha=2$.

Fig.4. The effects of parameters on the relative size $s_{\text{max}}$ of the largest polity. Each bar corresponds to a combination of four parameters: $\sigma$, $\theta$, $\tau$ and $L$. The values of $s_{\text{max}}$ are simultaneously reflected in the bar’s height, in the number shown next to it, and in the color of its top part. Other parameters are $S=5, \alpha=2$.

Fig. 5. Rank-size curves based on … [??]. Solid blue lines: the time average. Dashed blue lines: the time average plus minus one standard deviation. The green line gives the lognormal curve. Red line gives the rank-size curve at the final year of simulations. Parameters are as in Fig.3.
Tables

Table 2: Analysis of variance results on the contributions of the parameters and of their pairwise interactions into the total variance of statistics $s_{\text{max}}, \bar{c}, c_{\text{max}}, T$ and $\bar{P}$.

<table>
<thead>
<tr>
<th>parameters and their combinations</th>
<th>$s_{\text{max}}$</th>
<th>$\bar{c}$</th>
<th>$c_{\text{max}}$</th>
<th>$T$</th>
<th>$\bar{P}$</th>
</tr>
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<tbody>
<tr>
<td>S</td>
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<td>8.3</td>
<td>3.4</td>
<td>0.0</td>
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<tr>
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<td>33.6</td>
<td>34.9</td>
<td>19.9</td>
<td>5.4</td>
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<td>$\sigma$</td>
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<td>1.6</td>
<td>0.7</td>
<td>0.7</td>
<td>25.0</td>
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<td>$\theta$</td>
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<td>2.0</td>
<td>0.5</td>
<td>8.6</td>
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<td>$L$</td>
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<td>0.8</td>
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<tr>
<td>$\tau$</td>
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<td>$S*\alpha$</td>
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<td>0.0</td>
</tr>
<tr>
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<td>100.0</td>
<td>100.0</td>
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</tr>
<tr>
<td>( S )</td>
<td>System edge size</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>----------</td>
<td>------------------------------------------------------</td>
<td></td>
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<tr>
<td>( \alpha )</td>
<td>Scaling exponent (of the polity power to the probability of a win)</td>
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<tr>
<td>( \sigma )</td>
<td>Standard deviation of the baseline resource level</td>
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<tr>
<td>( \theta )</td>
<td>Tribute level</td>
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<td></td>
<td></td>
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<tr>
<td>( L )</td>
<td>Maximu span of control (i.e. the maximum number of subordinate communities)</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>( \tau )</td>
<td>The expected time in power of the paramount chief</td>
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<td>( s_{\text{max}} )</td>
<td>Relative size of the largest polity</td>
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<tr>
<td>( \bar{c} )</td>
<td>Mean complexity</td>
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<tr>
<td>( c_{\text{max}} )</td>
<td>Maximum complexity</td>
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<tr>
<td>( \rho )</td>
<td>Average centrality (i.e. the ratio of the power of the chief village and the one immediately below)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 2.
Figure 3:
Figure 4:
Figure 5.
Supplementary information

1. Sample movies with $\alpha = 1$ and $\alpha = 2$. Other parameters are at the midpoints of the ranges used ($S = 6; \sigma = 0; \theta = 0; 2; L = 6; \tau = 10$). The movies are currently available at http://neko.bio.utk.edu/~sergey/chiefdoms/chiefdoms.html

   The files are currently available at http://neko.bio.utk.edu/~sergey/chiefdoms/chiefdoms.html

3. Effects of parameters on the properties of the system
   The files are currently available at http://neko.bio.utk.edu/~sergey/chiefdoms/chiefdoms.html