Show all of your work as partial credit will be given.

1. Find a general solution to the differential equation using the method of variation of parameters

\[ y'' + 2y' + y = e^{-t} \ln t. \]

**Answers**

**Problem 1.**

First we consider the corresponding homogeneous equation

\[ y'' + 2y' + y = 0. \]

Its characteristic equation \( r^2 + 2r + 1 = 0 \) has a double root \( r = -1 \). Therefore,

\[ y_1 = e^{-t}, \quad y_2 = te^{-t}. \]

We write a particular solution as \( y_p = v_1y_1 + v_2y_2 \). The Wronskian is

\[ W = (e^{-t})'te^{-t} - e^{-t}(te^{-t})' = \cdots = -e^{-2t}. \]

Then

\[ v_1' = t \ln t, \quad v_2' = \ln t \]