

Show all of your work as partial credit will be given.

1. Find a general solution to the differential equation using the method of variation of parameters

$$y'' + 2y' + y = e^{-t} \ln t.$$

Answers

Problem 1.

First we consider the corresponding homogeneous equation

$$y'' + 2y' + y = 0.$$

Its characteristic equation $r^2 + 2r + 1 = 0$ has a double root $r = -1$. Therefore,

$$y_1 = e^{-t}, y_2 = te^{-t}.$$

We write a particular solution as $y_p = v_1 y_1 + v_2 y_2$. The Wronskian is

$$W = (e^{-t})' te^{-t} - e^{-t} (te^{-t})' = \dots = -e^{-2t}.$$

Then

$$v_1' = t \ln t, \quad v_2' = \ln t$$