

*Show all of your work as partial credit will be given.*

1. Using the separation of variables method explicitly solve the O.D.E. below for  $x$ :

$$\frac{dx}{dt} = 3x \cos t.$$

2. Using the separation of variables method explicitly solve the I.V.P. below for  $y$ :

$$\frac{dy}{dx} = 6x^2 e^{-y}, \quad y(1) = 0.$$

## Solutions

### Problem 1.

(i) First, we separate the variables:

$$\frac{dx}{x} = 3 \cos t \, dt.$$

(ii) Then we integrate both sides:

$$\int \frac{dx}{x} = \int 3 \cos t \, dt.$$

(iii) Therefore, all solutions satisfy to

$$\ln x = 3 \sin t + c,$$

where  $c$  is a constant, or

$$x = \exp(3 \sin t + c).$$

### Problem 2.

(i) First, we separate the variables:

$$e^y \, dy = 6x^2 \, dx.$$

(ii) Then we integrate both sides:

$$\int e^y \, dy = \int 6x^2 \, dx.$$

(iii) Therefore, all solutions satisfy to

$$e^y = 2x^3 + c,$$

where  $c$  is a constant, or

$$y = \ln(2x^3 + c).$$

From the initial conditions,

$$0 = \ln(2 + c).$$

Therefore,  $c = -1$ , and the solution of the initial value problem is

$$y = \ln(2x^3 - 1).$$