Show all of your work as partial credit will be given.

1. Using the separation of variables method explicitly solve the O.D.E. below for \( x \):
\[
\frac{dx}{dt} = 3x \cos t.
\]

2. Using the separation of variables method explicitly solve the I.V.P. below for \( y \):
\[
\frac{dy}{dx} = 6x^2 e^{-y}, \quad y(1) = 0.
\]

Solutions

Problem 1.
(i) First, we separate the variables:
\[
\frac{dx}{x} = 3 \cos t \, dt.
\]

(ii) Then we integrate both sides:
\[
\int \frac{dx}{x} = \int 3 \cos t \, dt.
\]

(iii) Therefore, all solutions satisfy to
\[
\ln x = 3 \sin t + c,
\]
where \( c \) is a constant, or
\[
x = \exp(3 \sin t + c).
\]

Problem 2.
(i) First, we separate the variables:
\[
e^y \, dy = 6x^2 \, dx.
\]

(ii) Then we integrate both sides:
\[
\int e^y \, dy = \int 6x^2 \, dx.
\]

(iii) Therefore, all solutions satisfy to
\[
e^y = 2x^3 + c,
\]
where \( c \) is a constant, or
\[
y = \ln(2x^3 + c).
\]

From the initial conditions,

\[
0 = \ln(2 + c).
\]

Therefore, \( c = -1 \), and the solution of the initial value problem is
\[
y = \ln(2x^3 - 1).
\]