Math 231: EXAM #2 **NAME:**
100 points. Each question is worth 25 points

Instructions: Read through entire exam before beginning to work. Do all your work on the sheets provided (please use only one side of each sheet). Show all of your work as partial credit will be given. **Books, notes, and calculators are not allowed.**

1. Find a general solution to

$$y'' - 2y' + y = 0.$$

2. Solve the initial value problem

$$y'' + 2y' + 17y = 0,\ y(0) = 1,\ y'(0) = -1.$$

3. Use the superposition principle and the method of undetermined coefficients to find a *general solution* to

$$y'' - 4y' + 5y = 5x - e^{2x}.$$

4. Use variation of parameters to find the general solution of

$$y'' + y = \frac{1}{\cos t}.$$

Solutions.

1. Characteristic equation $r^2 - 2r + 1 = 0$ has a double root $r = 1$ so $y = c_1 e^t + c_2 t e^t$.

2. Characteristic equation $r^2 + 2r + 17 = 0$ has a pair of complex roots: $r = -1 \mp 4i$ so that the general solution is $y = c_1 e^{-t} \cos 4t + c_2 e^{-t} \sin 4t$. Using initial conditions one finds that $c_1 = 1$ and c_2=0.

3. Characteristic equation $r^2 - 4r + 5 = 0$ has a pair of complex roots: $r = 2 \mp i$ so that the general solution of the homogeneous equation is $y_h = c_1 e^{2t} \cos t + c_2 e^{2t} \sin t$. The particular solution of a non-homogeneous equation $y'' - 4y' + 5y = 5x$ is $y_{p,1} = Ax + B$ with $A = 1, B = 4/5$. The particular solution of a non-homogeneous equation $y'' - 4y' + 5y = -e^{2x}$ is $y_{p,2} = Ae^{2x}$ with $A = -1$. The general solution is $y_h + y_{p,1} + y_{p,2}$.

4. Characteristic equation $r^2 + 1 = 0$ has a pair of complex roots: $r = \mp i$ so that the general solution of the homogeneous equation is $y_h = c_1 y_1 + c_2 y_2$ where $y_1 = \sin t$ and $y_2 = \cos t$. The corresponding Wronskian $W = 1$. A particular solution has a form $y_p = v_1 y_1 + v_2 y_2$. One finds that $v_1' = -1$ so that $v_1 = -t$ and that $v_2' = \sin t / \cos t$ so that $v_2 = -\ln(\cos t)$. Therefore the general solution is $c_1 \sin t + c_2 \cos t - t \sin t - \ln(\cos t) \cos t$.