Age structure in Population Models

References:

Caswell, H. 1989. Matrix Population Models. Sinauer, Sunderland, MA.

Charlesworth, B. 1980. Evolution in Age Structured Populations. Cambridge Univ. Press.

Keyfitz, N. 1968. Introduction to the Mathematics of Population. Addison-Wesley, Reading, MA.

Objectives of Demographic analyses of Populations:

1. Determine the effects of schedules of fertility and mortality on population structure and dynamics

2. Provide methods to allow comparisons of life histories between differing populations and species.

3. Provide the capability to estimate the impacts of alterations in vital statistics on popultion dynamics, e.g. both forecasting and prediction.

Terms:

Birth-flow population - births occur continuously over the time period of interest

Birth-pulse population - reproduction in a population in concentrated over a brief interval per time period of interest.

Census analysis - vertical method - use census information on a population obtained at least at two times to estimate vital statistics

Cohort analysis - horizontal method - follow a sample of individuals born at the same time til death to determine vital statistics

Forecasting - an attempt to predict what will happen

Maternity function - m(x) = mean number of offspring born per individual of age x per unit time - the age specific rate of reproduction

Projection - an attempt to predict what would happen given certain assumptions

Projection interval - the time step for a projection matrix

Projection matrix - a matrix having entries which estimate the survivorships from one age class to another and the fertilities of the age classes, which allows for projection of the numbers of individuals in each age class at time n+1 to be estimated from the numbers in each age class at time n.

Survivorship function - l(x) = Probability of survivorship from birth to age x

Vital statistics - birth and death rates of a population

Life Table analysis

This is a misnomer - a life table provides a schedule of the deaths within a population as a function of age by specifying the survivorship function l(x). Sometimes it is combined with the maternity function as well. Estimating these functions can be done a number of ways:

(a) Direct methods:

(i) cohort analysis in which one follows a cohort through time, estimating survivorship and fertility directly for this cohort

(ii) census analysis which requires some appropriate statistical methods to estimate survivorship as a function of age from observations of individuals in a population at two distinct times.

(b) Indirect methods - often necessary in non-laboratory cases, and all require additional estimates of fertility and survivorship for ages that can't be included in these studies

(i) assume the population is constant in size and has constant age-structure, from which one can estimate that l(x)/(Sum over all ages of l(x)) = the fraction of individuals in the population of age x

(ii) again assume the population is constant and has constant age ditribution and use a sample of the ages at death of individuals in the population as an estimate of l(x) - l(x+1) = frequency of individual who die while in age class x

(iii) again assuming constant age structure, mark a sample of individuals at birth (or some other age) and then recover as mnany marked dead individuals as posible to estimate distribution of age at death.

Basics of projection matrix approach:

Define an appropriate age class structure so all individuals aged from say 0 to L are in age class 1, those aged from L to 2L are in age class 2, etc. For simplicity we assume L=1 and that this is also the length of the projection interval.

Then with $n_i(t)$ = number of individuals in age class i at time t = 0,1,2,...

and

P_i = the survival probability of members of age class i then

$n_i(t+1)\_ = P_{i-1}\ n_{i-1}(t)$ for i = 2,3,4,... and t = 0,1,2,...

and if F_i = the fertilities = the number of age class 1 individuals at time t+1 per age class i individuals at time t, then

$n_1(t+1) = F_1\ n_1(t) + F_2\ n_2(t) + ...$

and these can all be expressed in matrix form as per the Leslie matrix

Determining estimates of the elements of the projection matrix is non-trivial and has been subject to a great deal of misunderstanding. There are quite different methods for birth-pulse versus birth-flow populations, and even within these there are several potentially different reasonable options. The key difficulty here arises because we are attempting to discretize what is in essence a continuous process, and the results make assumptions about the distribution of ages of individuals within each discrete age class.